Averkov, Gennadiy, Integer and discrete quantitative Helly numbers

I will present results obtained in a joint work with Bernardo González Merino, Matthias Schymura, Ingo Paschke and Stefan Weltge.
Let S be a discrete subset of $\mathbb{R}^n$ and define $c(S,k)$ as the smallest number with the property that if a finite family of convex sets has exactly k points of S in common, then at most $c(S,k)$ convex sets in this family already have exactly k points of S in common. For $S=\mathbb{Z}^n$, this number repeatedly appeared in different contexts as, for instance, optimization and geometry of numbers and, very recently, for general sets S, in the context of Helly and Tverberg theorems.
We give a useful description of $c(S,k)$ in terms of polytopes with vertices in S. Starting with this description, we answer several fundamental questions about $c(S,k)$. We provide the general upper bound $c(S,k) \leq \lfloor (k+1)/2 \rfloor (c(S,0)-2)+c(S,0)$ for every discrete S. For the integer lattice $S=\mathbb{Z}^n$, employing theory of lattice polytopes and the geometry of numbers, we solve the question on the asymptotic behavior by proving the estimate $c(\mathbb{Z}^n,k)=\Theta(k^{(n-1)/(n+1)})$ for every fixed n, and we compute the exact values of $c(\mathbb{Z}^n,k)$ for $k=0,\ldots,4$

Batyrev, Victor, Calabi-Yau varieties and lattice polytopes

The talk is devoted to some generalizations of reflexive polytopes in connection to Calabi-Yau varieties and Mirror Symmetry.

Di Rocco, Sandra, Polytopes for toric vector bundles

Many properties of a toric vector bundle can be explored by using associated lattice polytopes. I will explain the interplay between algebraic and convex geometry and the motivation for using polytopes. The talked will be partially based on joint work with K. Jabbusch and Greg Smith.

Higashitani, Akihiro, Characterizations for h^*-vectors of lattice polytopes

One of the most significant invariants of a lattice polytope is the Ehrhart polynomial encoding the number of lattice points contained in its integer dilation. By a suitable transformation, we obtain the h^*-polynomial (or h^*-vector, the sequence of its coefficients) which has many favorable properties. In this talk, after surveying the h^*-vector of a lattice polytope and its properties, we will focus on the characterization problem on the h^*-vector and provide some recent results.

Katthän, Lukas, Spanning lattice polytopes and the Uniform position principle

A lattice polytope is called spanning if its lattice points affinely span the ambient lattice. This property can be translated to a natural algebraic property of the Ehrhard ring of the polytope. In this talk, I will present recent joint work with Johannes Hofscheier and Benjamin Nill, where we use methods from commutative algebra and algebraic geometry to obtain new inequalities for the h^*-vector. This extends our previous work on the absence of inner zeros in the h^*-vector, as well as Hibi's inequality for polytopes with inner lattice points.

Kiritchenko, Valentina, Interactions of representation theory with lattice polytopes

Lattice polytopes arise naturally at the crossroads of representation theory and theory of Newton-Okounkov convex bodies. In my talk, I will describe several families of such polytopes from the viewpoint of convex geometry. In particular, I describe a convex geometric construction of Gelfand-Zetlin polytopes and Littelmann-Feigin-Fourier-Vinberg polytopes, which implies the equality of their Ehrhart polynomials.

Michalek, Mateusz, Are Ehrhart polynomials related to Riemann hypothesis?

In the talk we present relations among three classical mathematical objects: graphs, polytopes and polynomials. There are known constructions that associate to a graph a lattice polytope - the symmetric edge polytope. To each lattice polytope P one associates the Ehrhart polynomial that computes the number of lattice points in dilations of P. Furthermore, the roots of Ehrhart polynomials are also an object of intensive studies. There are many graphs for which these roots have a remarkable property: they lie on a line in complex plane with real part equal to $-1/2$. One of the first positive results was the case of the complete $(1, n)$-bipartite graphs (trees) proved independently by Kirschenhofer et al. and by Bump et al. In the latter this family of polynomials was studied in the context of the local Riemann hypothesis. We present recent results obtained jointly with Higashitani and Kummer proving several conjectures confirming when such polynomials have the correct root distribution. Our main new method relies on the use of interlacing polynomials - a technique generalizing orthogonal polynomials. Several open problems (probably easier than Riemann hypthesis) will be presented.

Pabiniak, Milena, Okounkov bodies and toric degenerations in symplectic geometry

A toric degeneration is a construction from algebraic geometry which allows us to "degenerate" a given projective manifold M to some (symplectic) toric variety X_0, i.e. form a flat family over $\mathbb{C}$ with a generic fiber M and a special fiber X_0. As observed by Harada and Kaveh, if M is symplectic then there exists an open dense subset of M and an open dense subset of X_0 which are symplectomorphic. This allows us to study symplectic invariants of M by studying X_0 which, as a toric variety, is usually much better understood. In certain nice situations the whole M is symplectomorphic to X_0 and the degeneration provides a symplectomorphism.
To construct a toric degeneration one needs a (very ample) line bundle over M and a (nice enough) valuation on the coordinate ring $\mathbb{C}(M)$. From these data one forms a semi-group S, and an Okounkov body P, which is a rational polytope if S is finitely generated. In that case the special (toric) fiber X_0 of the degeneration will be $\mathrm{Proj}(\mathbb{C}[S])$ and its normalization will be the normal toric variety associated to the polytope P.
In this talk I will describe the construction of toric degeneration and briefly discuss two of its applications in symplectic geometry.
1. To find lower bounds for the Gromov width of M, concentrating mostly on the case when M is a coadjoint orbit of a group G (based on projects with I. Halacheva, and with X. Fang and P. Littelmann). Here we will see interaction with representation theory: for appropriately chosen line bundle and valuation, the associated Okounkov body is equal to a string polytope, i.e. a polytope obtained from a string parametrization of a crystal basis of a representation of G.
2. To study questions of the form: given two (symplectic) Bott manifolds and an isomorphism F between their integral cohomology rings, sending $[\omega_1]$ to $[\omega_2]$, is there a diffeomorphism inducing F? (Based on a project with S. Tolman.)

Schaller, Karin, Stringy Chern classes of toric varieties, lattice polytopes, and the number 24

We give a combinatorial interpretation of the stringy Libgober-Wood identity in terms of generalized stringy Hodge numbers and intersection products of stringy Chern classes for arbitrary projective Q-Gorenstein toric varieties.
As a first application we derive a novel combinatorial identity relating reflexive polytopes of dimension $d \geq 4$ to the number 24. Our second application is motivated by computations of stringy invariants of non-degenerated hypersurfaces in 3-dimensional algebraic tori whose minimal models are K3-surfaces, giving rise to a combinatorial identity for the Euler number 24. Using combinatorial interpretations of the stringy E-function and the stringy Libgober-Wood identity, we show with purely combinatorial methods that this identity holds for any 3-dimensional lattice polytope containing exactly one interior lattice point.

Süß, Hendrik, Non-trivial Kaehler-Ricci solitons on singular del Pezzo surfaces

We are looking for the existence of certain canonical Kaehler metrics on Fano orbifolds, so called non-trivial Kaehler-Ricci solitons. In the surface case this question can be reduced to solving integral equations on the polytopes corresponding to toric degenerations of the original del Pezzo surfaces.
This is joint with Jacob Cable.

Tran, Bach, A combinatorial bound for k-normality of very ample lattice polytopes

For any very ample lattice polytope P, we will give a a bound of minimum integer k such that P is k-normal.

Tsuchiya, Akiyoshi, Gorenstein simplices with a given δ-polynomial

It is fashionable among the study on convex polytopes to classify the lattice polytopes with a given δ-polynomial. As a basic challenges toward the classification problem, we achieve the study on classifying lattice simplices with a given δ-polynomial of the form $1 + t^{k+1} + \cdots + t^{(v-1)(k+1)}$, where $k \geq 0$ and $v > 0$ are integers. The lattice polytope with the above δ-polynomial is necessarily Gorenstein. A complete classification is already known, when v is prime. In this talk, a complete classification will be performed, when v is either p^2 or pq, where p and q are prime integers with $p \neq q$. This talk is based on joint work with Takayuki Hibi and Koutarou Yoshida.

Tveiten, Ketil, Some ways to think about mutating Fano polygons

Reflexive polytopes are a special case of the class of *Fano polytopes*. I will talk about the operation of *mutation* on Fano polygons, which is important in Mirror Symmetry, and how this operation can be thought of in terms of quivers and dimer models.
