Corrigendum to: A MOSUM procedure for the estimation of multiple random change points

Claudia Kirch*

Otto-von-Guericke University Magdeburg, Institute for Mathematical Stochastics,

Postfach 4120, 39106 Magdeburg, Germany;

claudia.kirch@ovgu.de

June 19, 2018

Abstract

We correct a mistake in the proof and consequently statement of Theorem 2.3 in [1], where in b) the bound $\Lambda_n n^{1/2}/G$ must be replaced by $\max(\Lambda_n^{1/2}, \log n)\sqrt{\Lambda_n\, n}/G$.

1 Moving variance and long-run variance estimators

Consider the following moving estimator for the long-run covariance:

$$\hat{\tau}^2_{k,n} := \hat{\gamma}_k(0) + 2\sum_{h=1}^{\Lambda_n} \omega(h/\Lambda_n)\hat{\gamma}_k(h)$$

with autocovariance estimator

$$\begin{aligned}\hat{\gamma}_k(h) \quad := \quad & \frac{1}{2G}\sum_{i=k-G+1}^{k-h}(X_i - \overline{X}_{k-G+1,k})(X_{i+h} - \overline{X}_{k-G+1,k}) \\ & + \frac{1}{2G}\sum_{i=k+1}^{k+G-h}(X_i - \overline{X}_{k+1,k+G})(X_{i+h} - \overline{X}_{k+1,k+G}),\end{aligned}$$

bandwidth $\Lambda_n \geqslant 1$ and suitable weights ω, such as, for example, Bartlett or flat-top weights. For independent errors, we get a moving variance estimator by $\hat{\sigma}^2_{k,n} = \hat{\gamma}_k(0)$.

For convenience, we repeat the (corrected) statement of Theorem 2.3 b) in [1].

Theorem 1.1. *Let $X_i = \mu + \varepsilon_i$, $i = 1, \ldots, n$, $\{\varepsilon_i\}$ with $\mathrm{E}\,|\varepsilon_i|^4 < \infty$,*

$$\sup_{h\in\mathbb{Z}}\sum_{k\in\mathbb{Z}}\sum_{l\in\mathbb{Z}}|\nu(h,k,l)| < \infty, \tag{1.1}$$

where $\nu(h,r,s) = \operatorname{cov}(\varepsilon_1\varepsilon_{1+h}, \varepsilon_{1+r}\varepsilon_{1+s})$ and $\max_{1\leqslant k\leqslant n}\left|\sum_{i=1}^k \varepsilon_i\right| = O_P(\sqrt{n})$, which follows for example under Assumption A.1 b) in [1]. Then it holds:

a) If $n/G^2 = O(1)$*, we get*

$$\max_{G\leqslant k\leqslant n-G} |\hat{\sigma}^2_{k,n} - \sigma^2| \quad = \quad O_P\left(\frac{n^{1/2}}{G}\right).$$

b) If $\Lambda_n n/G^2 = O(1)$ *and the weights fulfill* $0 \leqslant w(x) \leqslant C$*, then*

$$\max_{G\leqslant k\leqslant n-G} |\hat{\tau}^2_{k,n} - \tau^2| \quad = \quad O_P\left(\max(\log n, \Lambda_n^{1/2})\,\frac{\sqrt{\Lambda_n n}}{G} + r_n\right).$$

where

$$r_n = \sum_{h\in\mathbb{Z}} |w(h/\Lambda_n) - 1|\,|\gamma(h)|.$$

Remark 1.1. As a consequence in Theorem 2.4 b) in [1] b) holds if $\frac{n\log^2 n}{G^2} = O(1)$ (which is related to (2.3) in [1]).

2 Corrected Proof of the Statement

The correction of the proof makes use of the following maximal inequality, which holds for any sequence of square integrable random variables Y_i, $i = 1,\dots,n$, and is a direct consequence of the generalization of the Rademacher-Menchoff maximal inequality given by Serfling [2]. The proof (of a slightly stronger statement replacing the absolute value on the right hand side by $(\cdot)_+$) can be found in [3], Lemma 2.2:

$$\mathrm{E}\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=1}^k Y_i\right|^2\right) \leqslant \left(\frac{\log 2n}{\log 2}\right)^2 \left[\sum_{i=1}^n \mathrm{E}\,Y_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^n |\mathrm{E}\,Y_iY_j|\right]. \tag{2.1}$$

Proof of Theorem 1.1. The last inequality in the first sequence of inequalities in the proof of Theorem 2.3 in [1] is not correct. In fact, by those argument only the statement $\leqslant C\frac{n\Lambda_n}{G\,\epsilon^2}$ can be obtained, which is too weak for the purpose of that paper. However, the methodology is sufficient to achieve the statement in (2.2) below.

First, note that by Kolmogorovs inequality (i.e. the Hájek -Renyi inequality with constant weights) it follows in the situation of (a), i.e. for independent errors,

$$\max_{0\leqslant k\leqslant n-G}\frac{1}{G}\left|\sum_{i=k+1}^{k+G}(\varepsilon_i^2 - \sigma^2)\right| \leqslant \frac{2}{G}\max_{0\leqslant k\leqslant n}\left|\sum_{i=1}^{k}(\varepsilon_i^2 - \sigma^2)\right| = O_P\left(\frac{\sqrt{n}}{G}\right),$$

so that the proof can be concluded as in [1].

For b) it holds

$$\max_{0\leqslant k\leqslant n-G}\left|\frac{1}{G}\sum_{h=0}^{\Lambda_n}\omega(h/\Lambda_n)\sum_{i=k+1}^{k+G-\Lambda_n}(\varepsilon_i\varepsilon_{i+h} - \gamma(h))\right|$$
$$\leqslant \frac{2}{G}\max_{1\leqslant k\leqslant n-\Lambda_n}\left|\sum_{i=1}^{k}\sum_{h=0}^{\Lambda_n}\omega(h/\Lambda_n)\,(\varepsilon_i\varepsilon_{i+h} - \gamma(h))\right| = O_P\left(\log n\,\frac{\sqrt{\Lambda_n\, n}}{G}\right),$$

where the rate follows from Markovs inequality in combination with (2.1) by putting $Z_{i,n} = \sum_{h=0}^{\Lambda_n} \omega(h/\Lambda_n)\left(\varepsilon_i\varepsilon_{i+h} - \gamma(h)\right)$. To elaborate:

$$\begin{aligned}
\mathrm{E}\max_{1\leqslant k\leqslant n-\Lambda_n}\left|\sum_{i=1}^{k} Z_{i,n}\right|^2 &\leqslant C(\log n)^2\Bigg[\sum_{i=1}^{n-\Lambda_n}\sum_h\sum_b \left|\mathrm{E}\left(\varepsilon_i\varepsilon_{i+h}-\gamma(h)\right)\left(\varepsilon_i\varepsilon_{i+b}-\gamma(b)\right)\right| \\
&\qquad + \sum_{i=1}^{n-\Lambda_n-1}\sum_{j=i+1}^{n-\Lambda_n}\sum_h\sum_b \left|\mathrm{E}\left(\varepsilon_i\varepsilon_{i+h}-\gamma(h)\right)\left(\varepsilon_j\varepsilon_{j+b}-\gamma(b)\right)\right|\Bigg] \\
&\leqslant C(\log n)^2\sum_{i=1}^{n-1}\sum_{j=i}^{n}\sum_h\sum_b |\nu(h,j-i,j-i+b)| \leqslant C(\log n)^2\, n\Lambda_n \sup_h\sum_k\sum_l |\nu(h,k,l)| \\
&\leqslant C(\log n)^2\, n\Lambda_n,
\end{aligned}$$

where C is a generic constant that can differ at every occurence. Furthermore, along the lines of the original proof as given in [1], we get

$$\max_{0\leqslant k\leqslant n-G}\left|\frac{1}{G}\sum_{h=0}^{\Lambda_n-1}\omega(h/\Lambda_n)\sum_{i=k+G-\Lambda_n+1}^{k+G-h}\left(\varepsilon_i\varepsilon_{i+h}-\gamma(h)\right)\right| = O_P\left(\frac{\Lambda_n\sqrt{n}}{G}\right). \quad (2.2)$$

The proof can now be concluded as in [1]. ■

References

[1] Birte Eichinger and Claudia Kirch. A mosum procedure for the estimation of multiple random change points. *Bernoulli*, 24(1):526–564, 2018.

[2] Robert J Serfling. Moment inequalities for the maximum cumulative sum. *The Annals of Mathematical Statistics*, pages 1227–1234, 1970.

[3] Soo Hak Sung. Maximal inequalities for dependent random variables and applications. *Journal of Inequalities and Applications*, 2008. DOI: 10.1155/2008/598319.