

Otto-von-Guericke-Universität Magdeburg  
Fakultät für Mathematik

Auf Einladung des Institutes für Algebra und Geometrie spricht

Herr Dr. Tan Nhat Tran

(Ruhr-Universität Bochum)

über das Thema

### **Characteristic and Ehrhart quasi-polynomials for root systems**

**Der Vortrag findet hybrid statt.**

Entweder Zoom Meeting (ID 971 4945 5855, passcode 490213) oder G03-223

**Zeit:** Dienstag, 30. November 2021, 13.00 Uhr

Zu diesem Vortrag laden wir alle Interessierten herzlich ein.

Dr. Yuri Santos Rego

**Abstract:** In enumerative combinatorics, counting the size of a set depending upon a positive integer  $q$  often gives rise to polynomials in  $q$  (e.g., the chromatic polynomial of a graph), and sometimes quasi-polynomials. Generally speaking, a quasi-polynomial is a generalization of polynomials, of which the coefficients may not come from a ring but instead are periodic functions with integral period. One of the most classical examples is the Ehrhart quasi-polynomial that counts the number of integral points in the  $q$ -fold dilation of a rational polytope. In the theory of hyperplane arrangements, a quasi-polynomial appears when we count the size of the complement of an integral hyperplane arrangement modulo  $q$  - the characteristic quasi-polynomial due to Kamiya-Takemura-Terao. My talk will be divided into three small sections. In the first section, we define the characteristic and Ehrhart quasi-polynomials and give basic facts. Then we recall the concept of period collapse in Ehrhart quasi-polynomials, and present analogous results for characteristic quasi-polynomials. This section is based on a recent joint work with A. Higashitani (Osaka) and M. Yoshinaga (Hokkaido). In the second section, we introduce the notion of (Worpitzky-)compatible subsets of an irreducible root system together with an associated Eulerian polynomial, which brings the characteristic and Ehrhart quasi-polynomials into one formula. Inspired by the fact that the  $h$ -polynomial of the (Steinberg torus) Coxeter complex always equals the (affine)  $W$ -Eulerian polynomial, we propose an open question regarding our Eulerian polynomial. This part is based on a joint work with A. U. Ashraf (Karachi) and M. Yoshinaga. In the third section, we continue the discussion in the second section with a focus on type  $A$  root systems. We show that the compatible graphic arrangements are characterized by cocomparability graphs, and discuss an open problem posed by Brenti regarding the log-concavity of an Eulerian polynomial of graphs. This section is based on a joint work with A. Tsuchiya (Tokyo).