Otto-von-Guericke-Universität Magdeburg
Fakultät für Mathematik

Auf Einladung des Institutes für Algebra und Geometrie spricht

Herr Christopher Voll

(Universität Bielefeld)

über das Thema

Groups, graphs, and hypergraphs - counting conjugacy classes of p-groups and averaging generic matrices with support constraints

Zoom-Koordinaten: https://ovgu.zoom.us/j/95199662620
Meeting ID: 951 9966 2620 / Passcode: 461614

Zeit: Dienstag, 22. Dezember 2020, 14.00 Uhr

Zu diesem Vortrag laden wir alle Interessierten herzlich ein.

Prof. Dr. Petra Schwer

Abstract: A fundamental invariant of a finite group is its total number of conjugacy classes or, equivalently, of irreducible complex characters. Computing this class numberïs, in general, a difficult problem. Several long-standing conjectures in the theory of finite p-groups concern the distribution of class numbers in natural families of groups. For instance, the class numbers of the groups of uppertriangular matrices over finite fields are famously conjectured (by G. Higman) to be given by polynomials in the fields' cardinalities.

The story I will tell is about joint work with Tobias Rossmann (NUI Galway). Together, we develop a theory of average sizes of kernels of matrices with support constraints defined in terms of graphs and hypergraphs. We apply this theory to derive strong polynomiality results about the variation of class numbers of finite 'graphical groups', viz. certain finite p-groups canonically associated with graphs. These results are in marked contrast with deep algebro-geometric results by Belkale and Brosnan on the rank distribution of symmetric matrices over finite fields with such 'graphical' support constraints: the precise rank distributions are, in a precise technical sense, arbitrarily wild; the average rank distributions are, as we show, quite tame, viz. described by polynomials.

Methodologically our project draws together ideas from the theory of groups and graphs, but also toric geometry and p-adic integration. My talk, however, will assume little more than bachelor-level abstract algebra. https://arxiv.org/abs/1908.09589