

Otto-von-Guericke-Universität Magdeburg
Fakultät für Mathematik

Auf Einladung des Institutes für Mathematische Stochastik spricht

Herr Prof. Dr. Pierre Bellec

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über das Thema

Degrees-of-freedom in high-dimensional statistics

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Zu diesem Vortrag sind alle Interessierten herzlich eingeladen.

Abstract:

In classical statistics, degrees-of-freedom adjustments arise naturally in the construction of unbiased estimators; for instance in linear regression with p parameters and $p \ll n$, the residual sum of squares divided by $(n-p)$ estimates the noise level without bias. I will present in this talk two examples of degrees-of-freedom adjustments in high dimensions (i.e., when $n \gg p$), and explain the mathematics that justify applying these degrees-of-freedom adjustments for estimators commonly used to analyze high-dimensional data.

A well-understood degrees-of-freedom adjustment appears in Stein's Unbiased Risk Estimate (SURE) to construct an unbiased estimate of the mean square risk of almost any estimator $\tilde{\mu}$; here the divergence of $\tilde{\mu}$ plays the role of degrees-of-freedom or the estimator. Thanks to Stein's formula, not only unbiased estimates can be constructed for the risk of $\tilde{\mu}$, but also for the risk of SURE itself (SURE for SURE): a simple unbiased estimate provides information about the squared distance between SURE and the squared estimation error of $\tilde{\mu}$, again involving degrees-of-freedom adjustments.

A second area where degrees-of-freedom appear is the construction of confidence intervals. A novel analysis reveals that degrees-of-freedom adjustments play a major role in de-biasing methodologies to construct confidence intervals in high-dimension. We will see that in sparse linear regression for the Lasso for Gaussian designs, existing de-biasing schemes need to be modified with an adjustment that accounts for the degrees-of-freedom of the Lasso. This degrees-of-freedom adjustment is necessary for statistical efficiency in the regime $s \gg n^{2/3}$. Here, the necessity of degrees-of-freedom adjustment is explained using the interpolation path between Gaussian vectors typically used to derive Slepian's lemma.