

Otto-von-Guericke-Universität Magdeburg  
Fakultät für Mathematik

Auf Einladung des Institutes für Mathematische Stochastik spricht

**Herr Prof. Dr. Pierre Bellec**

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über das Thema

## **Degrees-of-freedom in high-dimensional statistics**

**Ort:** Otto-von-Guericke-Universität, G18-401

**Zeit:** 9. Januar 2020, 13:15 Uhr

Zu diesem Vortrag sind alle Interessierten herzlich eingeladen.

### **Abstract:**

In classical statistics, degrees-of-freedom adjustments arise naturally in the construction of unbiased estimators; for instance in linear regression with  $p$  parameters and  $p \ll n$ , the residual sum of squares divided by  $(n-p)$  estimates the noise level without bias. I will present in this talk two examples of degrees-of-freedom adjustments in high dimensions (i.e., when  $n \gg p$ ), and explain the mathematics that justify applying these degrees-of-freedom adjustments for estimators commonly used to analyze high-dimensional data.

A well-understood degrees-of-freedom adjustment appears in Stein's Unbiased Risk Estimate (SURE) to construct an unbiased estimate of the mean square risk of almost any estimator  $\tilde{\mu}$ ; here the divergence of  $\tilde{\mu}$  plays the role of degrees-of-freedom or the estimator. Thanks to Stein's formula, not only unbiased estimates can be constructed for the risk of  $\tilde{\mu}$ , but also for the risk of SURE itself (SURE for SURE): a simple unbiased estimate provides information about the squared distance between SURE and the squared estimation error of  $\tilde{\mu}$ , again involving degrees-of-freedom adjustments.

A second area where degrees-of-freedom appear is the construction of confidence intervals. A novel analysis reveals that degrees-of-freedom adjustments play a major role in de-biasing methodologies to construct confidence intervals in high-dimension. We will see that in sparse linear regression for the Lasso for Gaussian designs, existing de-biasing schemes need to be modified with an adjustment that accounts for the degrees-of-freedom of the Lasso. This degrees-of-freedom adjustment is necessary for statistical efficiency in the regime  $s \gg n^{2/3}$ . Here, the necessity of degrees-of-freedom adjustment is explained using the interpolation path between Gaussian vectors typically used to derive Slepian's lemma.